

Closing *Thurs*: HW_6A,6B (7.4, 7.5)

Closing next *Wed*: HW_7A,7B (7.5, 7.7, 7.8)

Note: Friday is a university holiday (no class)

Exam 2 is next Thursday (Nov. 16th)

Covers 6.4, 6.5, 7.1-7.5, 7.7, 7.8

Entry Task: Evaluate

$$\int \frac{x^2 + 6}{x^2 + 3x + 2} dx$$

Example: Evaluate

$$\int \frac{x^2 - x + 6}{x^3 + 3x} dx$$

Example:

$$\int \frac{x}{x^2 + 4x + 5} dx$$

How to integrate

- A. Look for simplifications/substitutions
- B. Products/Logs/Inverse Trig → BY PARTS
 - Sin/Cos/Tan/Sec combos → TRIG
 - Quadratic (under a radical) → TRIG SUB
 - Rational Function → PART. FRAC.
- C. If nothing seems to work, substitution.
(u = inside, u = $\sqrt{\quad}$, u = trig, u = e^x)

Examples:

1. $\int e^{\sqrt{x}} dx$

$$2. \int \frac{\cos(x)}{4 - \sin^2(x)} dx$$

$$3. \int \frac{3}{x - 2\sqrt{x}} dx$$

$$4. \int e^x \cos(e^x) \sin^3(e^x) dx$$

How would you *start* these?

1. $\int \tan^3(x) \sec(x) dx$

2. $\int x^2 \ln(x) dx$

3. $\int x \sqrt{5 - x^2} dx$

4. $\int \frac{\sqrt{x^2 - 1}}{x^2} dx$

5. $\int \frac{x^2 + 1}{x^2 - 2x - 3} dx$

6. $\int x \tan^{-1}(x) dx$

7. $\int \frac{dx}{\sqrt{4x^2 + 8x - 12}} dx$

7.7 Approximating Integrals (Preview)

Despite our best efforts in 7.1-7.5, the vast majority of integrals can NOT be done with any of our methods.

So we usually have to approximate!

In this section we add two more approximation methods that are slightly more accurate. We already know left, right, and midpoint methods (but I included them below for completeness).

To approximate $\int_a^b f(x)dx$

1. Pick $n =$ number of subdivisions.

$$\text{Compute } \Delta x = \frac{b-a}{n}.$$

2. Label the tick marks: $x_i = a + i\Delta x$
3. Use an approximation method:

$$L_n = \Delta x [f(x_0) + f(x_1) + \cdots + f(x_{n-1})] \quad (\text{Left endpoint})$$

$$R_n = \Delta x [f(x_1) + f(x_2) + \cdots + f(x_n)] \quad (\text{Right endpoint})$$

$$M_n = \Delta x [f(\bar{x}_1) + f(\bar{x}_2) + \cdots + f(\bar{x}_n)] \quad (\text{Midpoint})$$

New - Trapezoid Rule: (all the “middle terms” are multiplied by 2)

$$T_n = \frac{1}{2} \Delta x [f(x_0) + 2f(x_1) + \cdots + 2f(x_{n-1}) + f(x_n)]$$

New - Simpson's Rule: n must be even! (Alternating multiplying middle terms by 4 and 2)

$$S_n = \frac{1}{3} \Delta x [f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + \cdots + 2f(x_{n-2}) + 4f(x_{n-1}) + f(x_n)]$$

Example: (Note: none of our methods can integrate this)

Estimate

$$\int_0^3 \sqrt{100 - x^3} dx$$

Here is how to use the left, right, midpoint and trapezoid rules with $n = 3$ subdivisions:

$$L_3 = (1) \left[\sqrt{100 - (0)^3} + \sqrt{100 - (1)^3} + \sqrt{100 - (2)^3} \right] \approx 29.5415$$

$$R_3 = (1) \left[\sqrt{100 - (1)^3} + \sqrt{100 - (2)^3} + \sqrt{100 - (3)^3} \right] \approx 28.0855$$

$$M_3 = (1) \left[\sqrt{100 - (0.5)^3} + \sqrt{100 - (1.5)^3} + \sqrt{100 - (2.5)^3} \right] \approx 29.0091$$

$$T_3 = \frac{1}{2} (1) \left[\sqrt{100 - (0)^3} + 2\sqrt{100 - (1)^3} + 2\sqrt{100 - (2)^3} + \sqrt{100 - (3)^3} \right] \approx 28.8135$$

Here is how to use Simpson's rule with $n = 6$ subdivisions (n has to be even to use this method):

$$S_6 = \frac{1}{3} \cdot \frac{1}{2} \left[\sqrt{100 - (0)^3} + 4\sqrt{100 - (0.5)^3} + 2\sqrt{100 - (1)^3} + 4\sqrt{100 - (1.5)^3} \right. \\ \left. + 2\sqrt{100 - (2)^3} + 4\sqrt{100 - (2.5)^3} + \sqrt{100 - (3)^3} \right] \approx 28.9441$$

“Actual” Value (to 8 places after the decimal): 28.94418784